Anti-aliasing Prefilter (6B)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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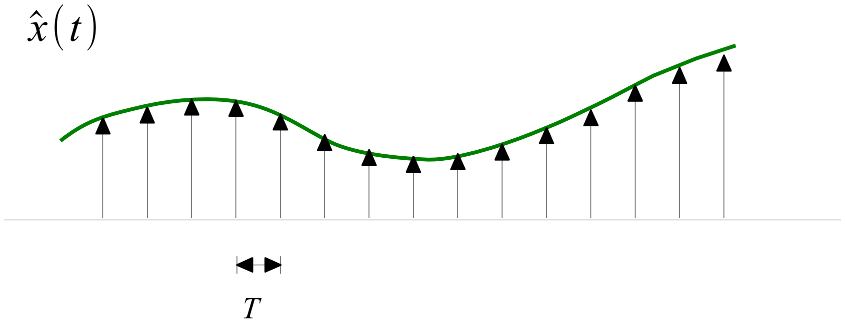
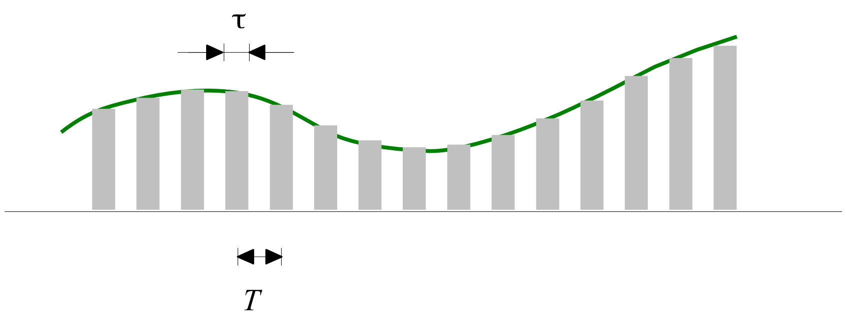
Sampler

Ideal Sampling



Practical Sampling

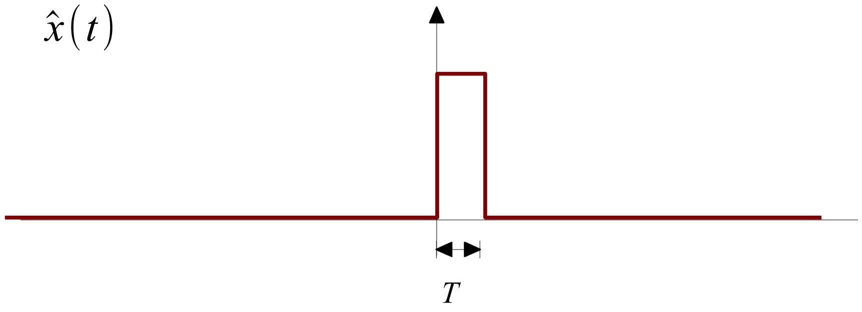


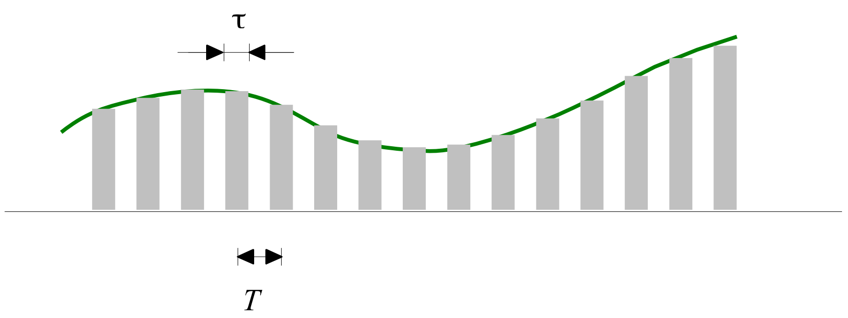
$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$\hat{x}(t) \approx \sum_{n=-\infty}^{+\infty} x(nT)\,p(t-nT)$$

$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t)\,e^{-j2\pi f t}\,dt$$

Zero Order Hold (ZOH)





Square Wave CTFT

Continuous Time Fourier **Series**

$$C_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt \quad \Longleftrightarrow \quad x(t) = \sum_{n=0}^{\infty} C_k\, e^{+jk\omega_0 t}$$

$$C_k = \frac{1}{T_0}\int_{-T_0/2}^{+T_0/2} x_{T_0}(t)\, e^{-jk\omega_0 t}\, dt$$

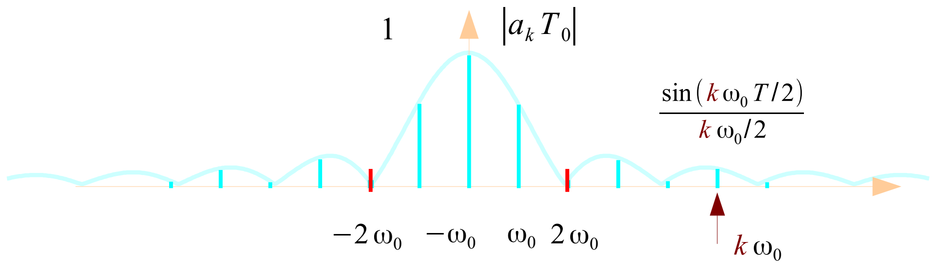
$$\omega_0 = \frac{2\pi}{T_0}$$

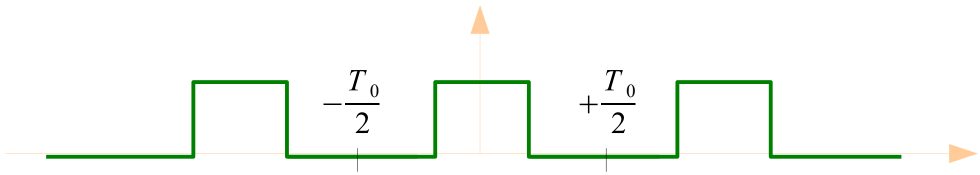
Fundamental Frequency

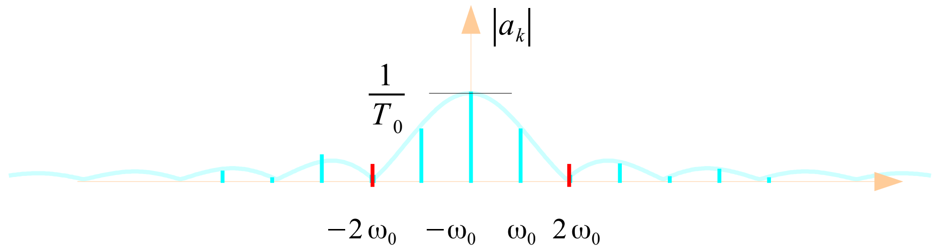
$$C_k T_0 = \int_{-T_0/2}^{+T_0/2} x_{T_0}(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \int_{-T_0/2}^{+T_0/2} e^{-jk\omega_0 t}\, dt = \left[\frac{-1}{jk\omega_0} e^{-jk\omega_0 t}\right]_{-T/2}^{+T/2}$$

$$= -\frac{e^{-jk\omega_0 T/2} - e^{+jk\omega_0 T/2}}{jk\omega_0} = \frac{\sin(k\omega_0 T/2)}{k\omega_0/2}$$

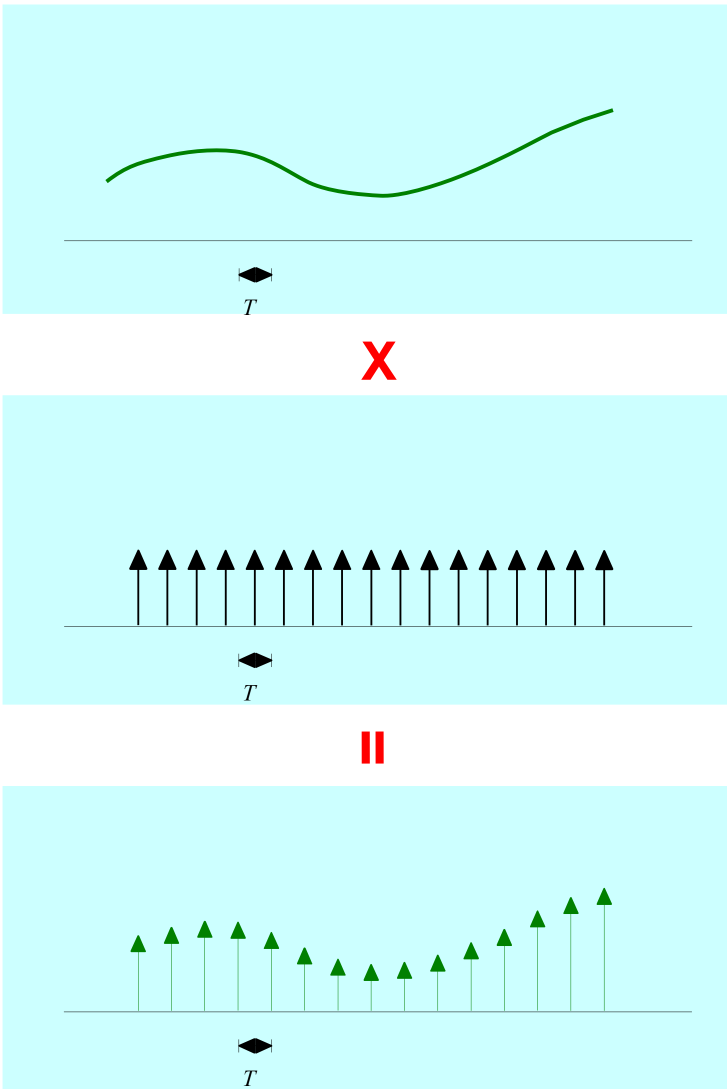




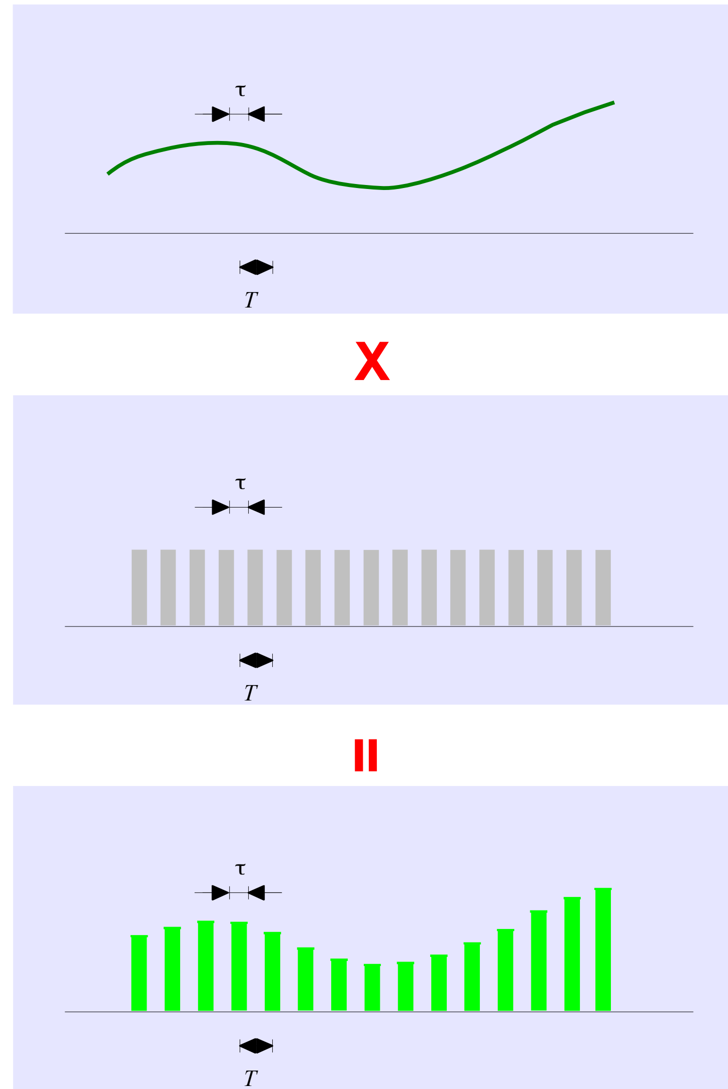


Sampling (1)

Ideal Sampling

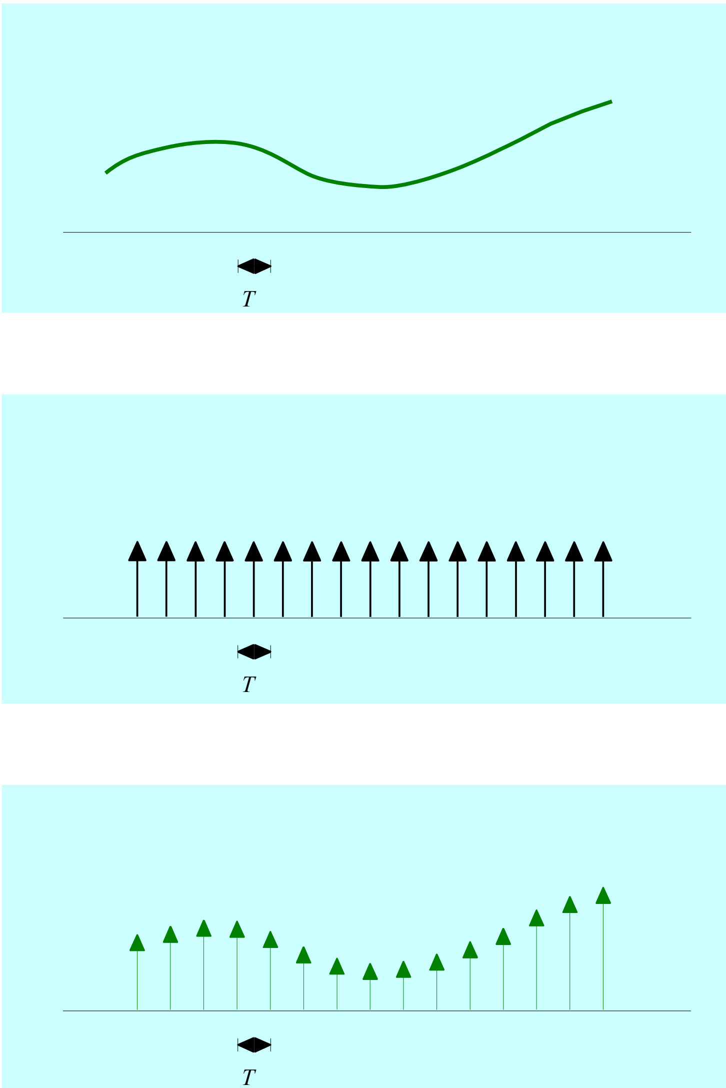


Practical Sampling

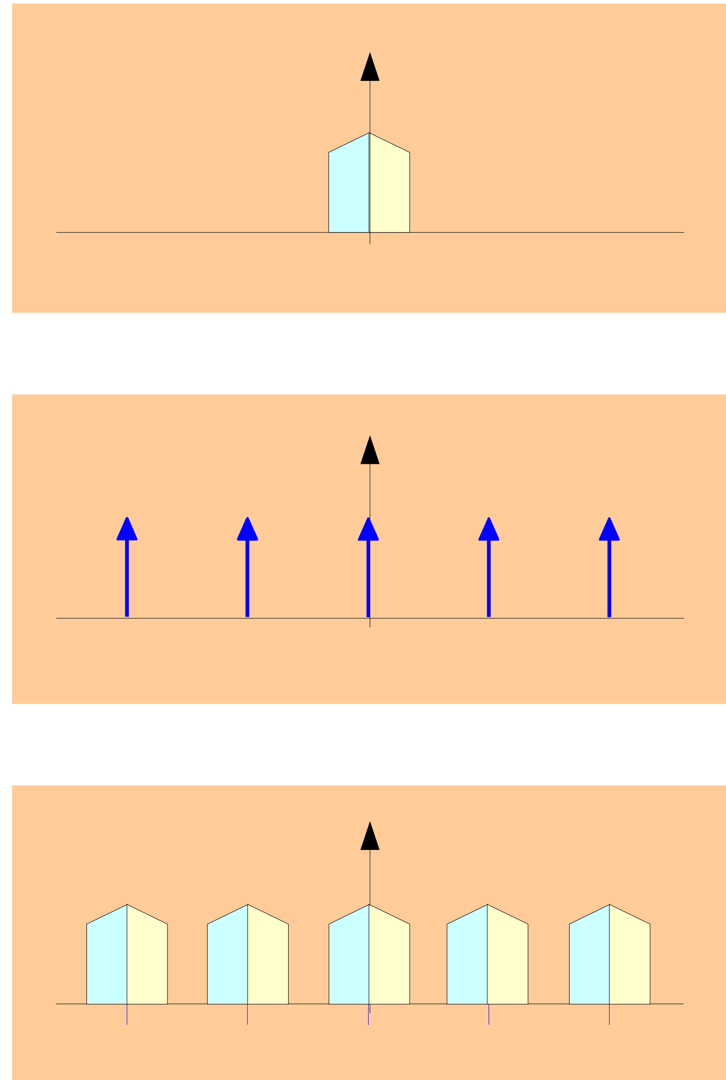


Sampling (2)

Ideal Sampling

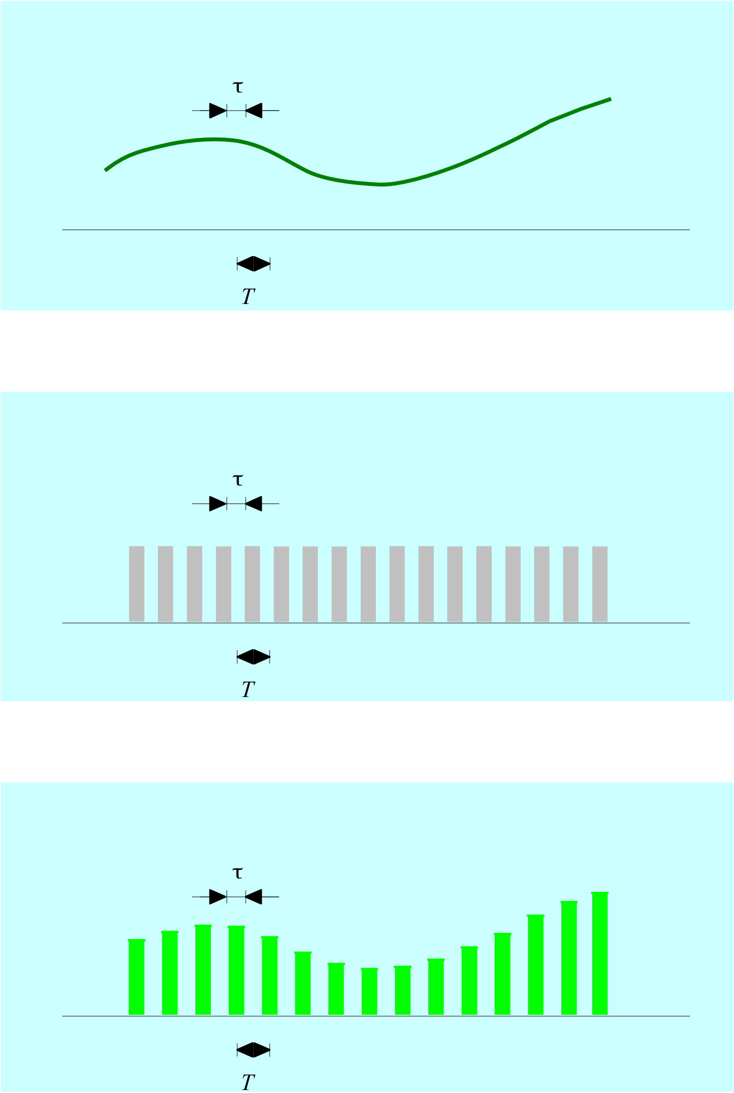


Frequency Domain

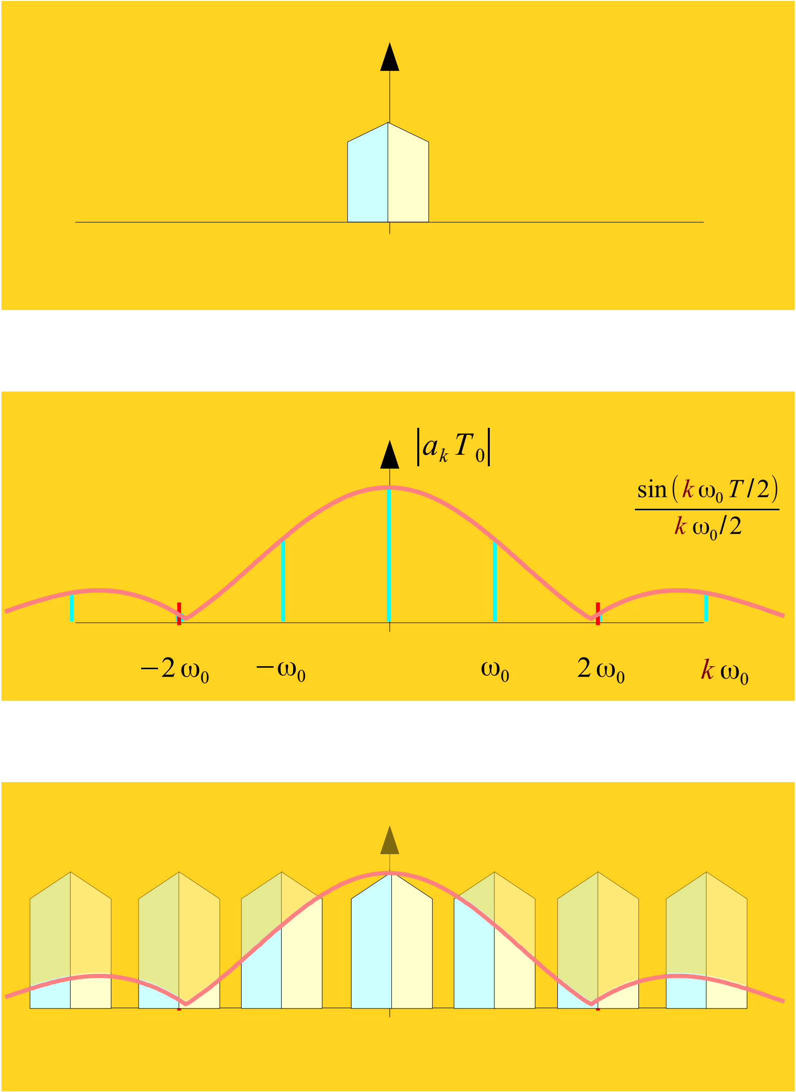


Sampling (3)

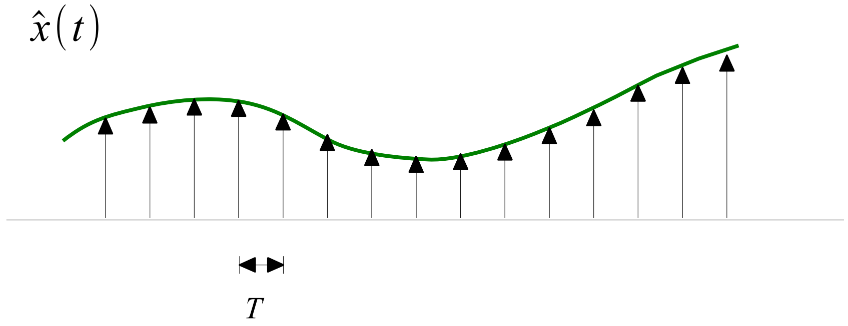
Practical Sampling



Frequency Domain



Discrete Time Fourier Transform



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

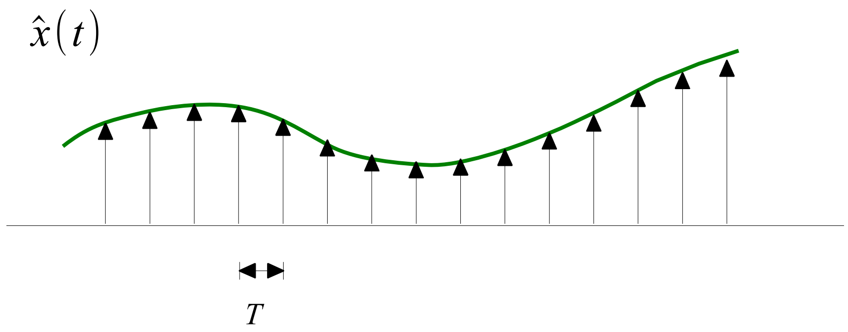
$$\hat{X}(f) = \int_{-\infty}^{+\infty} \hat{x}(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)\, e^{-j2\pi f t}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

Fourier Series



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\,e^{-j2\pi f T n}\,dt$$

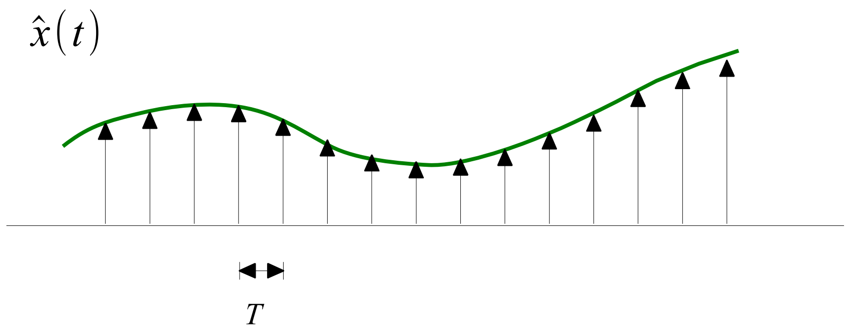
$$x(nT) = \frac{1}{f_s}\int_{-f_s/2}^{+f_s/2} \hat{X}(f)\,e^{+j2\pi f T n}\,df$$

$$= \int_{-\pi}^{+\pi} \hat{X}(\omega)\,e^{+j\omega n}\,\frac{d\omega}{2\pi}$$

$$\omega = 2\pi f / f_s$$

$$\frac{d\omega}{2\pi} = \frac{df}{f_s}$$

Numerical Approximation



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

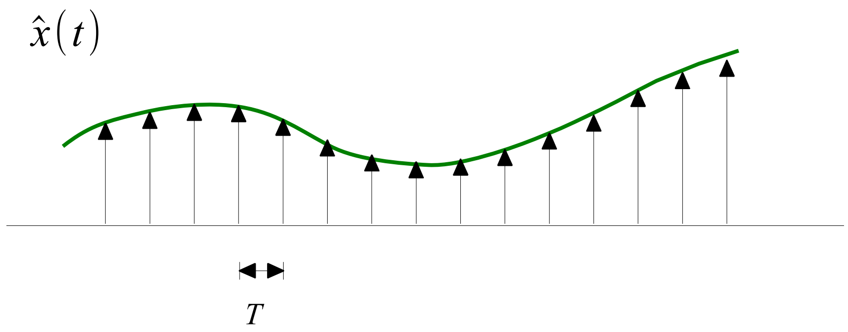
$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{+j2\pi f t}\, dt$$

$$\approx \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n} \cdot T$$

$$X(f) \approx T\,\hat{X}(f)$$

$$X(f) = \lim_{T \to 0} T\,\hat{X}(f)$$

Spectrum Replication (1)



$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$= x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT) = x(t)\,s(t)$$

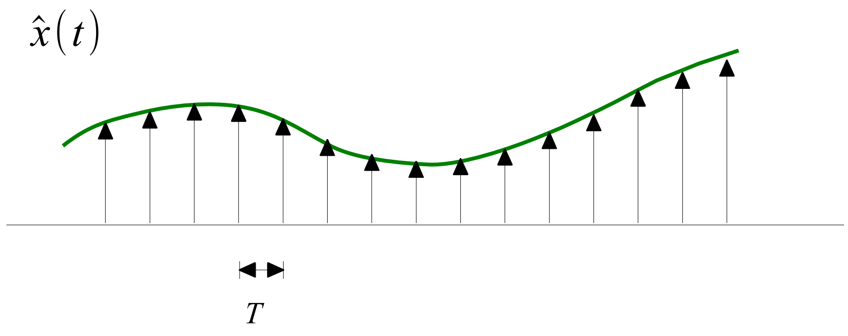
$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT) = \frac{1}{T} \sum_{m=-Infintiy}^{+\infty} e^{+j2\pi m f_s t}$$

$$\hat{x}(t) = x(t)\,s(t) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} x(t)\, e^{+j2\pi m f_s t}$$

$$\hat{X}(f) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} X(f-m f_s)$$

Spectrum Replication (2)



$$\hat{X}(f) = \sum_{n=-\infty}^{+\infty} x(nT)\, e^{-j2\pi f T n}\, dt$$

$$\hat{x}(t) = \sum_{n=-\infty}^{+\infty} x(nT)\,\delta(t-nT)$$

$$= x(t) \sum_{n=-\infty}^{+\infty} \delta(t-nT) = x(t)\,s(t)$$

$$\hat{X}(f) = \int_{-\infty}^{+\infty} X(f-f')S(f')\, df'$$

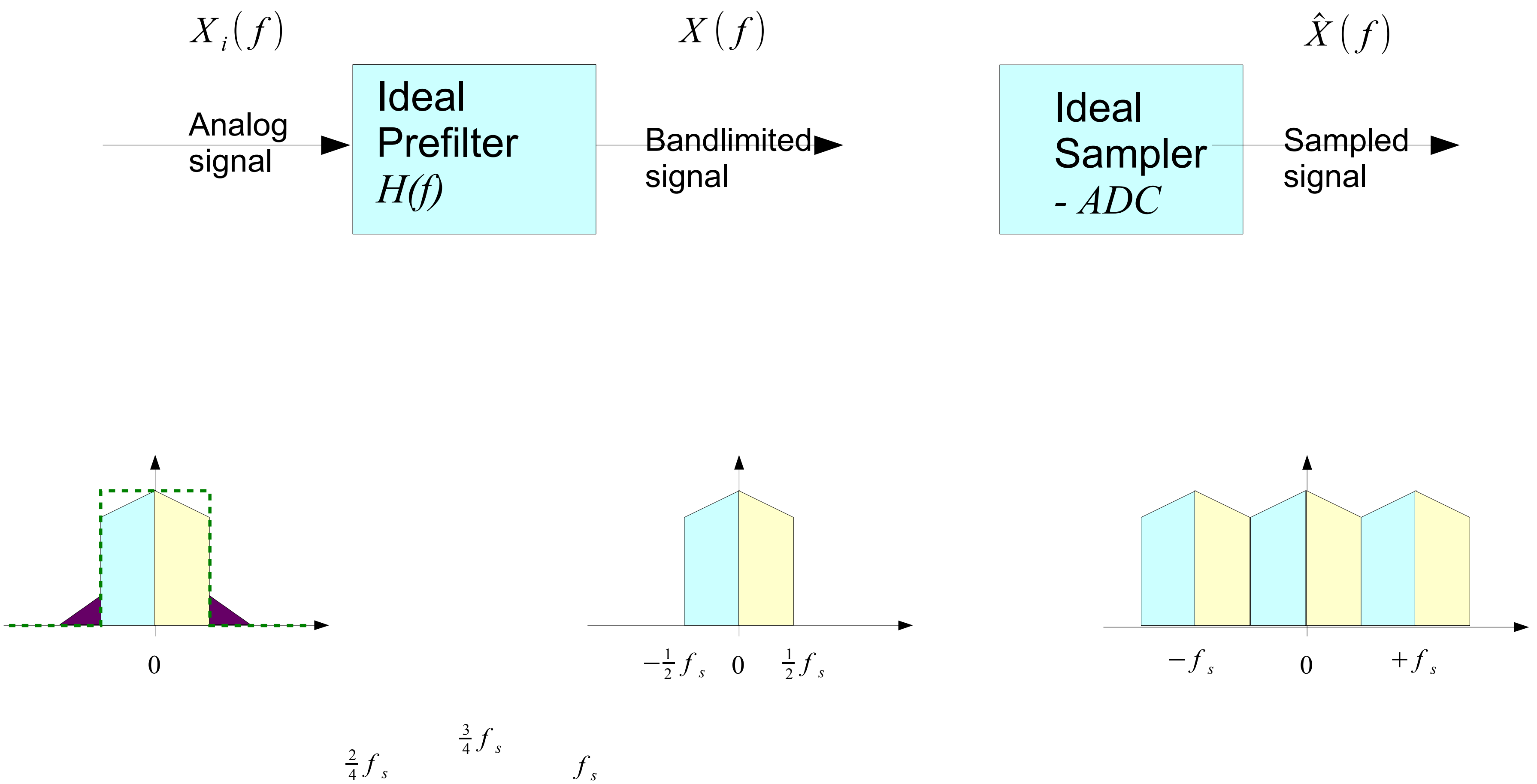
$$= \frac{1}{T} \sum_{m=-\infty}^{+\infty} \int_{-\infty}^{+\infty} X(f-f')\delta(f'-mf_s)\, df'$$

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t-nT) = \frac{1}{T} \sum_{m=-Infintiy}^{+\infty} e^{+j2\pi m f_s t}$$

$$S(f) = \frac{1}{T} \sum_{m=-\infty}^{+\infty} \delta(f-mf_s)$$

$$\hat{x}(t) = x(t)\,s(t) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} x(t)\, e^{+j2\pi m f_s t}$$

$$\hat{X}(f) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} X(f-mf_s)$$



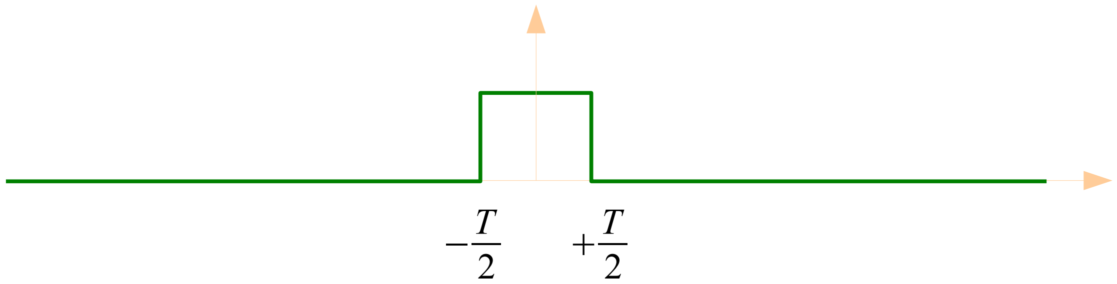
$X_i(f)$
Analog signal
Ideal Prefilter $H(f)$
$X(f)$
Bandlimited signal
Ideal Sampler - ADC
$\hat{X}(f)$
Sampled signal
0
$-\frac{1}{2}f_s$
0
$\frac{1}{2}f_s$
$-f_s$
0
$+f_s$
$\frac{2}{4}f_s$
$\frac{3}{4}f_s$
f_s

CTFT and CTFS (1)

Continuous Time Fourier <u>Transform</u>

Aperiodic Continuous Time Signal

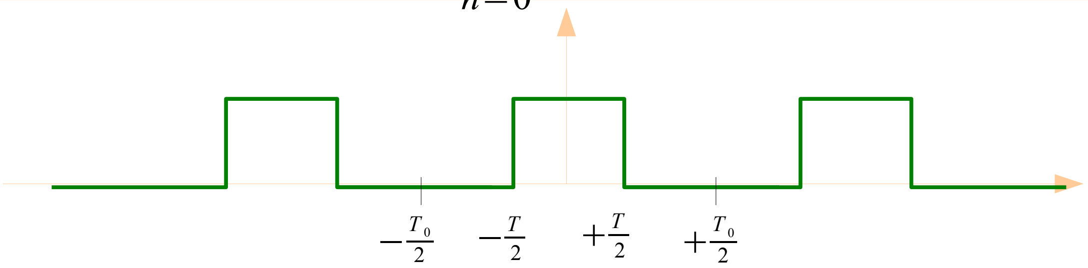
$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt \quad \Longleftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$



Continuous Time Fourier <u>Series</u>

Periodic Continuous Time Signal

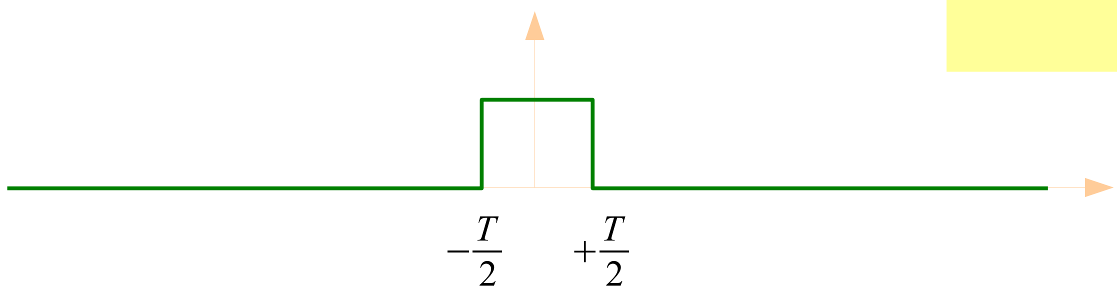
$$C_k = \frac{1}{T} \int_{0}^{T} x(t)\, e^{-jk\omega_0 t}\, dt \quad \Longleftrightarrow \quad x(t) = \sum_{n=0}^{\infty} C_k\, e^{+jk\omega_0 t}$$



CTFT and CTFS (2)

Aperiodic Continuous Time Signal

Continuous Time Fourier Transform



$$x(t)$$

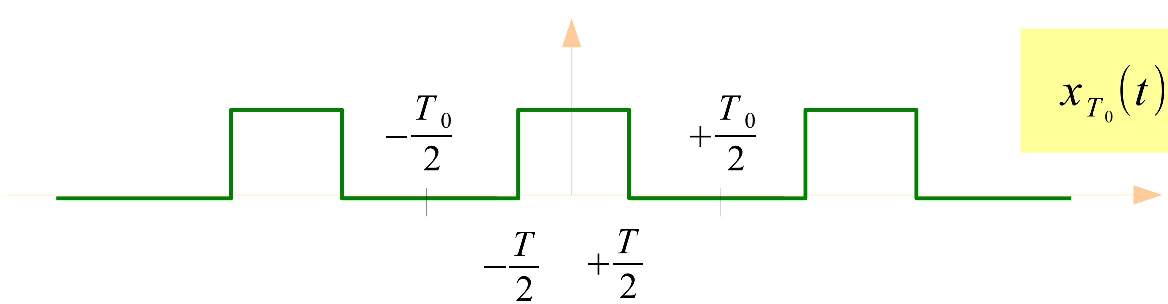
As $T_0 \to \infty$,

$$x_{T_0}(t) \to x(t)$$

$$\omega_0 = \frac{2\pi}{T_0} \to 0$$

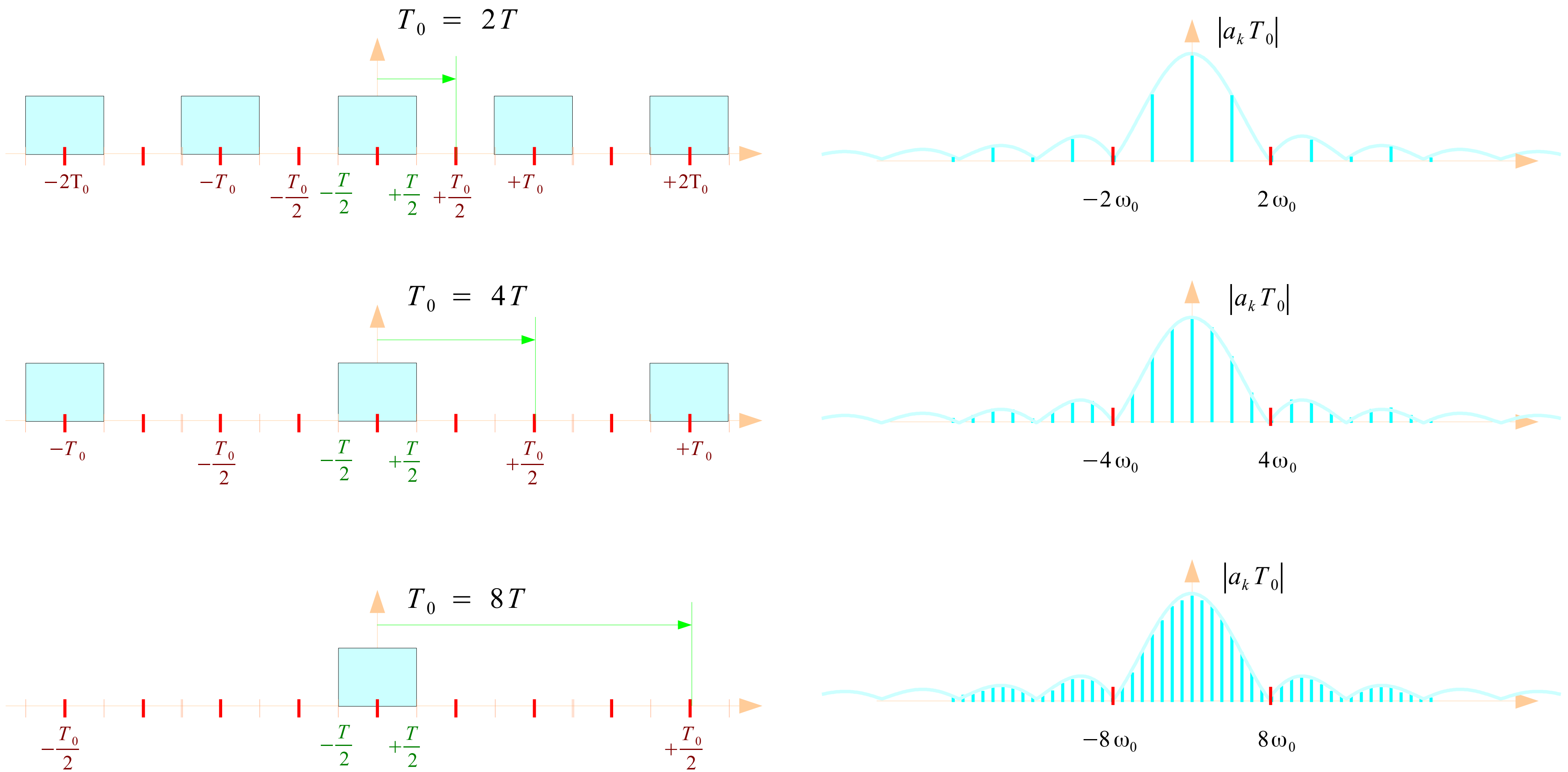
Periodic Continuous Time Signal

Continuous Time Fourier Series



$$x_{T_0}(t) = \sum_{n=-\infty}^{+\infty} x(t - nT_0)$$

CTFT and CTFS (3)



References

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[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
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www.ece.rutgers.edu/~orfanidi/intro2sp